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This report summarizes the research performed on various problems related to visibility probabilities, when the fields of vision are obscurred by random elements. Several Technical Reports have been published. The paper on the three-dimensional problem was published in the Naval Research Logistics Quarterly (1988). The paper on the multi-observer multi-target visibility probabilities is under revision for the Naval Research Logistics Quarterly. The Principal Investigator has also written a monograph on the subject based on a workshop manual. The research problems, the Table of Contents for the monograph and the introductory sections of the Technical Reports follow hereby are contained in this report.

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Visibility Probabilities And Distributions Of Random Visibility Measures In Two and Three Dimensional Spaces Having Random Obscuring Elements

Final Report

by

Shelemyahu Zacks, Principal Investigator

October 27, 1988

Contract DAAG29-84-K-0191

with the U.S. Army Research Office

Center for Statistics

Quality Control and Design

State University of New York,

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The present report summarizes the research performed under Contract DAAG29-84-K-0191 on various problems related to visibility probabilities, when the fields of vision are obscurred by random elements. Several Technical Reports have been published. The paper on the three-dimensional problem was published in the *Naval Research Logistics Quarterly* (1988). The paper on the multi-observer multi-target visibility probabilities is under revision for the Naval Research Logistics Quarterly. The Principal Investigator has also written a monograph on the subject based on a workshop manual. The research problems, the Table of Contents for the monograph and the introductory sections of the Technical Reports follow hereby.

Two papers were published during the contract period.

1. The Visibility Of Stationary And Moving Targets In The Plane Subject To Poisson Field Of Shadowing Elements, *J. Appl. Prob.*, **22**: 776-786 (1985).
2. Visibility Probabilities on Line Segments In Three-Dimensional Spaces Subject to Random Poisson Fields of Obscuring Spheres, *Naval Research Log. Quart.*, **35**: 555-569 (1988).

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THE VISIBILITY OF STATIONARY AND MOVING TARGETS IN THE PLANE SUBJECT TO A POISSON FIELD OF SHADOWING ELEMENTS

M. YADIN,* *Technion — Israel Institute of Technology*
S. ZACKS,** *State University of New York at Binghamton*

Abstract

A methodology for an analytical derivation of visibility probabilities of n stationary target points in the plane is developed for the case when shadows are cast by a Poisson random field of obscuring elements. In addition, formulae for the moments of a measure of the total proportional visibility along a star-shaped curve are given.

VISIBILITY PROBABILITY, MEASURE OF PROPORTIONAL VISIBILITY, POISSON RANDOM SHADOWING PROCESS

0. Introduction

The present paper discusses problems of visibility of targets through random fields of obscuring elements (trees, bushes, clouds, etc.). The problems dealt with are of a stochastic nature. The exact number, location and dimensions of the obscuring elements are unknown. A probability model is formulated concerning these variables. Given such a probability model, it is required to determine probabilities of certain events and distributions of certain random variables. To illustrate some of the problems that can be solved by the methodology developed, consider the following examples.

Example 1. *Visibility of stationary targets.* An observer is placed at a given location in a forest, in order to detect specified targets (vehicles, animals, etc.). Due to the random location of the trees it is important to determine the probabilities that individual targets are observed and the distribution of the number of targets observed. For this purpose one has to determine the

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Visibility Probabilities on Line Segments in Three-Dimensional Spaces Subject to Random Poisson Fields of Obscuring Spheres*

M. Yadin and S. Zacks
Technion, Israel Institute of Technology and State University of New York at Binghamton

The methodology of determining simultaneous visibility probabilities and points on line segments is extended to problems of three-dimensional spaces, with Poisson random fields of obscuring spheres. Required functions are derived analytically and a numerical example is given for a special case of a standard Poisson field, with uniform distribution of sphere diameters.

1. INTRODUCTION

The present article studies a three-dimensional shadowing problem, and provides an algorithm for the computation of visibility probabilities, for cases where the target is a straight line segment, or points on a line segment, which is parallel to a layer of shadowing objects. The results given here can be generalized to target curves in three-dimensional space, provided they lie in a plane containing the observation point **O**. As we show in Section 5, the theory developed here can be applied, for example, to determine visibility probabilities for a pilot approaching a runway under cloudy conditions. Many other applications are possible for military and civil-type problems.

Consider a problem of visibility obstruction, when an observer (source of light) is located at the origin **O** and the obscuring (shadowing) elements are randomly dispersed between the origin and the target curve. A straight line segment $\mathscr{C}$ is specified in a three-dimensional Euclidean space. The obscuring elements are modeled as spheres whose centers are randomly dispersed within a layer bounded between two planes parallel to $\mathscr{C}$, between the origin and the target curve. It is further assumed that the centers of spheres are dispersed according to a homogeneous Poisson process in the layer. The assumption about the homogeneity of the Poisson process is not crucial and can be relaxed. The radii of spheres are i.i.d. random variables having a distribution independent of the locations of the centers. The problem is to determine simultaneous visibility probabilities for n arbitrarily specified points on $\mathscr{C}$. Moments of integrated measures of visibility on $\mathscr{C}$ can be determined, as shown in [2], by integrating these visibility probabilities functions.

The methodology of the present study is to reduce the three-dimensional shadowing problem to a two-dimensional one, on the plane $\mathcal{M}$ containing the line segment $\mathscr{C}$ and the origin **O**. The treatment of the shadowing problem on $\mathcal{M}$ can follow the methodology developed in our previous studies [4,5]. There are, however, several complications.

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VISIBILITY PROBABILITIES AND DISTRIBUTIONS OF RELATED MEASURES OF VISIBILITY IN THE PLANE UNDER POISSON RANDOM FIELDS, I: ONE OBSERVATION POINT

by

Shelemyahu Zacks
Professor of Mathematical Sciences and
Director, Center for Statistics,
Quality Control and Design

Technical Monograph No. 1
March, 1986

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Discretization Of A Semi-Markov Shadowing Process

by

M. Yadin and S. Zacks

Technical Report No. 2

October 15, 1986

Prepared Under Contract DAAG29-84-K-0191 with the U.S. Army Research Office

S. Zacks, Principal Investigator

CENTER FOR STATISTICS, QUALITY CONTROL AND DESIGN
State University of New York
University Center at Binghamton
Binghamton, NY 13901

Discretization Of A Semi-Markov Shadowing Process*

by

M. Yadin and S. Zacks

Technion, Israel Institute of Technology and

State University of New York at Binghamton

ABSTRACT

A shadowing process in the plane is studied, with respect to a single source of light and a Poisson field of shadowing elements, located between the source of light and a target curve. N subsegments of length δ are considered within a specified portion of the target curve. Random measures of visibility and random shadow weights on the target curves are approximated by corresponding discrete random variables, defined for the N subsegments. Algorithms are provided for the computation of the distribution functions of the approximating discrete random variables.

Key words: Shadowing processes, Semi-Markov processes, discrete approximation, Poisson fields, measure of visibility.

*Research supported in Contract DAAG29-84-K-0191 with the U.S. Army Research Office.

1. *INTRODUCTION.*

Consider a ***source of light*** and a linear ***path*** in a plane. Random objects are scattered between the ***source of light*** and the ***path***, casting shadows on portions of it.

The *Shadowing Model* was originally introduced by Chernoff and Dally [1]. In their paper they assumed that the shadowing objects are *disks* having centers which are distributed in a rectangle between the *source of light* and the *path*, according to a two dimensional *Poisson Process*, and their diameters are represented by i.i.d. random variables. The paper applies queueing theoretical techniques to derive the conditional distributions of the lengths of the shadowed and the unshadowed segments of the *path*.

The shadowed and unshadowed segments of the *path* are analogous to busy and idle periods in certain queueing processes with Markov arrivals. Like in these processes, the derivation of the distribution of the lengths of unshadowed segments (which are similar to idle periods) is straightforward. Chernoff's and Dally's paper concentrates on the distributions of shadowed segments, and provide some integral equations for these distributions. The solution of these equations requires generally lengthy numerical procedures.

Yadin and Zacks [2, 3] introduced the concept of *Random Measures of Visibility*, which are defined as the total length of the unshadowed subsegments of a specified segment of the *path*, or the proportion of the segment which is in the light. In these papers the distributions of *Measures of Visibility* were approximated by a mixture of a two points distribution on $\{0,1\}$ and a beta distribution on $(0,1)$. These *mixed beta* distributions were obtained by equating the first three moments of the distributions of *Measures of Visibility* to the first three moments of a mixed-beta distribution. In [2] the *path* was assumed to be a segment of a circle, centered at the origin at which the *source of light* was located. The centers of the disks were assumed to be distributed within an annular region in the circle. The results were extended in [3] and [4], which considered more general *paths* and *fields of obscuring objects*. The methodology was extended to a three dimensional model in [5]. Certain applications were discussed in [7].

The main objective of the present paper is to replace the *mixed beta* distributions by discrete distributions which approximate the required distributions. More specifically, we approximate the distribution of a measure of visibility, by the distribution of a discrete random variable, being the number of the proportion of the unshadowed subsegments, which are properly located on some segment of the *path*.

The shadowing model and the approximated *measures of visibility* and lengths of shadowed segments are formally defined in section 2. The approximating distributions can be given in terms of *probabilities of elementary events*, which are the events signifying which of the N subsegments are completely unshadowed among the set of all N subsegments. These probabilities can be obtained from *probabilities of simultaneous visibility*, namely the probabilities that all subsegments in a specified sets are unshadowed. Simultaneous visibility probabilities were determined for various cases in [4]. Appendix A presents the methodology needed for the derivations of *probabilities of simultaneous visibility*.

The accuracy of the discrete approximation to the distributions considered depends on the number of subsegments. Assuming a set of N subsegments, one has to compute 2^N *probabilities of elementary events* from the same number of *probabilities of simultaneous visibility*.

Any algorithm which is based on the *probabilities of simultaneous visibility* is necessarily non polynomial, thus is limited to a relatively small number of points, and may therefore yield insufficient accuracy. Fortunately, the distributions of the approximated *measures of visibility* can be obtained by a polynomial algorithm which is discussed in section 4.

The results of a numerical example are presented and discussed in section 5. These results illustrate the advantages and limitations of the algorithms developed in the present paper with respect to accuracy, computer time and memory requirements.

References

[1] Chernoff, H. and Daly, J.F. (1957) The Distribution of Shadows, *J. of Math. and Mechanics* 6: 567-584.

[2] Yadin, M. and Zacks, S. (1982) Random Coverage of A Circle with Applications to a Shadowing Problem, *J. App. Prob.*, 19:562-577.

[3] Yadin, M. and Zacks, S. (1984) The Distribution of The Random Lighted Portion Of A Curve In A Plane Shadowed By A Poisson Random Field Of Obstacles *Statistical Signal Processing*, E.J. Wegman and J.G. Smith, Ed. Marcel Dekker, New York.

[4] Yadin, M. and Zacks, S. (1985) The Visibility of Stationary and Moving Targets In The Plane Subject To A Poisson Field of Shadowing Elements, *J. App. Prob.*, 22:776-786.

[5] Yadin, M. and Zacks, S. (1986) The Distributions of Measures of Visibility On Line Segments In Three Dimensional Spaces Under Poisson Shadowing Processes, (Revision) *Technical Report No. 3*, Contract DAAG29-84-K-0191 with the ARO, Center for Statistics, SUNY-Binghamton.

[6] Yadin, M. and Zacks, S. (1986) Multi-Observer Multi-Target Visibility Probabilities For Poisson Shadowing Processes In The Plane. *Technical Report No. 4*, Contract DAAG29-84-K-0191 with the ARO, Center for Statistics, SUNY-Binghamton.

[7] Zacks, S. (1986) Visibility Probabilities And Distributions Of Related Measures Of Visibility In The Plane Under Poisson Random Fields I: One Observation Point, *Technical Report No. 1*, Contract DAAG29-84-K-0191 with the ARO, Center for Statistics, SUNY-Binghamton.

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MULTI-OBSERVER MULTI-TARGET VISIBILITY
PROBABILITIES FOR POISSON SHADOWING PROCESSES
IN THE PLANE

by

M. Yadin and S. Zacks

TECHNICAL REPORT NO. 4

October 15, 1986

Prepared under contract
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S. Zacks Principal Investigator

CENTER FOR STATISTICS, QUALITY CONTROL AND DESIGN
University Center of the State University
of New York at Binghamton

MULTI-OBSERVER MULTI-TARGET VISIBILITY PROBABILITIES FOR POISSON SHADOWING PROCESSES IN THE PLANE*

by

M. Yadin and S. Zacks
Technion, Israel Institute of Technology and
State University of New York at Binghamton

ABSTRACT

Visibility Probabilities are studied for Poisson shadowing processes in the plane, for stationary targets and several observation points. Efficient algorithms are developed for the computation of simultaneous visibility probabilities, and for the probabilities of all elementary events.

Key Words: *Poisson shadowing processes, stationary targets, multi-observer, random fields, visibility probabilities*

* Supported by Contract DAAG29-84-K-0191 with the U. S. Army Research Office.

1. Introduction.

In the present paper we consider problems associated with the visibility of targets from several observation points, subject to interference by a random field of obscuring elements. This study is an extension of previous ones [1, 2, 3, 4], which deal with visibility problems associated with a single observation point. We consider a random field of obscuring elements, which are centered in a region located between the observation points and the targets. The targets are stationary points in the plane. Dynamic targets will be considered in another paper. The objective of the present study is to develop efficient algorithms for the computation of visibility probabilities. For example, if there are three observation points and three targets, there are nine possible lines of sight, connecting observation points with targets. These lines of sight might be intersected by obscuring elements. We say that a target $\underset{\sim}{T}$ is observable from $\underset{\sim}{O}$ if the corresponding line of sight is not intersected. An elementary event is a statement which specifies which lines of sight are intersected and which are not. In the above example there are $2^9 = 512$ such elementary events. The problem is to determine the probabilities of elementary events, as functions of properties of the random field of obscuring elements - its geometrical structure and its stochastic characteristics.

In Section 2 we present the shadowing model, the characteristics of the field, and define a few random variables of interest. For example, the number of targets, N_i, observable simultaneously from the i-th observation point; or the number of observation points M_j from which the j-th target can be simultaneously observed; etc.

Distributions related to these variables are discussed. In Section 3 we define simultaneous visibility probabilities of sets of observation points/targets; and the probabilities of elementary events. We show how vectors of elementary probabilities can be obtained from vectors of simultaneous visibility probabilities by linear transformations, using Hadamard matrices of order 2^n. In Section 4 we develop the algorithm for the computation of simultaneous visibility probabilities. Section 5 presents a geometrical development of formulae required for the determination of certain functions defined in Section 4. Finally, in Section 6 we provide a numerical example, related to three observation points and three targets. In this example we illustrate the application of the procedures developed in the present study to answer various questions of interest. We tabulate also the joint probability distribution function (P.D.F.) of (N_1,N_2,N_3), and the three joint marginals of pairs of random variables. These joint distributions reflect the type of stochastic dependence between these random variables, and show that the regular correlations are very low. This result is, however, a function of the particular location of the observation points and of the targets. By changing the location coordinates of these points the correlations in the above distributions may increase or decrease. Different indexes of stochastic dependence can be developed for different problems under consideration. The simplest index of dependence can be based on the simultaneous visibility probability of two or more targets from a given observation point. Another index of dependence, could be the correlation between N_i and N_j $(i \neq j)$, or M_i and M_j $(i \neq j)$. The algorithms developed in the present study can serve as basic tools for the computation of such indexes.

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Visibility Probabilities On Line Segments In Three Dimensional Spaces Subject To Random Poisson Fields Of Obscuring Spheres.*

by

M. Yadin and S. Zacks
Technion, Israel Institute of Technology
and State University of New York at Binghamton

Abstract

The methodology of determining simultaneous visibility probabilitites and points on line segments is extended to problems of three dimensional spaces, with Poisson random fields of obscuring spheres. Required functions are derived analytically and a numerical example is given for a special case of a standard Poisson field, with uniform distribution of sphere diameters.

Key Words: Poisson Shadowing Processes, Lines of Sight; Visibility Probabilities; Measures of Visibility; Moments of Visibility.

*Research supported by contract DAAG29-84-K-0191 with the U.S. Army Research Office.

1. *Introduction.*

The present paper studies a three-dimensional shadowing problem, and provides an algorithm for the computation of visibility probabilities, for cases where the target is a straight line segment, or points on a line segment, which is parallel to a layer of shadowing objects. The results given here can be generalized to target curves in three-dimensional space, provided they lie in a plane containing the observation point $\underset{\sim}{O}$. As we show in Section 5, the theory developed here can be applied, for example, to determine visibility probabilities for a pilot approaching a runway under cloudy conditions. Many other applications are possible for military and civil type problems.

Consider a problem of visibility obstruction, when an observer (source of light) is located at the origin, $\underset{\sim}{O}$, and the obscuring (shadowing) elements are randomly dispersed between the origin and the target curve. A straight line segment, C, is specified in a three-dimensional Euclidean space. The obscuring elements are modeled as spheres whose centers are randomly dispersed within a layer bounded between two planes parallel to C, between the origin and the target curve. It is further assumed that the centers of spheres are dispersed according to a homogeneous Poisson process in the layer. The assumption about the homogeneity of the Poisson process is not crucial and can be relaxed. The radii of spheres are i.i.d. random variables having a distribution independent of the locations of the centers. The problem is to determine simultaneous visibility probabilities for n arbitrarily specified points on C. Moments of integrated measures of visibility on C can be determined, as shown in [2], by integrating these visibility probabilities functions.

The methodology of the present study is to reduce the three-dimensional shadowing problem to a two-dimensional one, on the plane $\mathcal{M}$ containing the line segment C and the origin $\underset{\sim}{O}$. The treatment of the shadowing problem on $\mathcal{M}$ can follow the methodology developed in our previous studies [3, 4]. There are, however, several complications. Even if the Poisson field of sphere centers within the layer is homogeneous, the projection of the field into $\mathcal{M}$ generally yields a non-homogeneous field of disks. The radii of disks on $\mathcal{M}$

depend on the location of the corresponding centers in the layer, the length of the sphere radii and their distances for $\mathcal{M}$. Suppose that the distributions of radii of spheres are supported by a finite interval $(0, b)$, where b is sufficiently small, so that neither the origin $\underset{\sim}{O}$ nor the target C can be covered by a random sphere. Obviously, spheres having centers with distance from $\mathcal{M}$ greater than b can be ignored.

In order to avoid computational difficulties created by the non-homogeneity of the field on $\mathcal{M}$, we partition the layer into a large number of thin sublayers. The projection of the random fields restricted to these infinitesimal sublayers on $\mathcal{M}$ are strips of almost homogeneous sub-fields.

Given n specified points on C, consider the rays originating at $\underset{\sim}{O}$ and passing through these points. The n points are simultaneously visibile (in the light) if none of the random disks on $\mathcal{M}$ intersects any one of these rays. Thus, if $E(n)$ designates the expected number of disks on $\mathcal{M}$, which intersect at least one of the specified rays, then the probability of simultaneous visibility is $\exp(-E(n))$. The main contribution of the present paper is in developing the formulae for computing $E(n)$. For this purpose we present in Section 2 the three dimensional model and the basic geometry. In Section 3 we discuss the reduction to a two-dimensional shadowing process. In Section 4 we present the formula for computing simultaneous visibility probabilities. Analytical derivation of some of the functions specified in Section 4 is given in the Appendix. In Section 5 we provide an explicit solution for the case of i.i.d. sphere radii having a uniform distribution on $(0, b)$.

In order to assist the reader in implementing the methodology of the present paper, we highlight below the main steps required for the computation of visibility probabilities. A FORTRAN program according to which the example of Section 5 has been computed can be furnished upon request.

STEP 1: Given $\underset{\sim}{O}$ and C, specify the parallel planes (U^*) and (W^*) which constitute the boundaries to the layer of random spheres; the intensity, λ of the Poisson field in the layer and the distribution function $F(\cdot)$ of radii of spheres.

STEP 2: Determine the plane C^* containing C, which is parallel to $(\mathcal{W}^*)$; and the distances u^*, w^*, r^* from $\underset{\smile}{O}$ of these planes.

STEP 3: Determine the projections $\underset{\smile}{O}^*$ and $\underset{\smile}{O}''$ of $\underset{\smile}{O}$ on C^* and on C, respectively. Let d^* be the distance of $\underset{\smile}{O}''$ from $\underset{\smile}{O}^*$. The inclination angle of $\mathcal{M}$ is $\phi = \arctan(d^*/r^*)$.

STEP 4: Determine $\beta = b^* \tan(\phi) = b \cdot d^*/r^*$;

STEP 5: Prepare a subroutine to compute the function $K(s,t)$ according to formulae (4.6), (4.7) and (A.13). [The coordinates s are measured along C relative to $\underset{\smile}{O}''$.]

STEP 6: Compute simultaneous visibility probabilities, according to (4.1)-(4.3) and (4.9)-(4.10).

2. *The Three Dimensional Shadowing Model.*

Consider a source of light located at a point $\underset{\smile}{O}$ (the origin), and an arbitrary straight line, C, in a three-dimensional Euclidean space. A layer of random spheres is located between two planes $\mathcal{U}^*$, $\mathcal{W}^*$, which are parallel to the line C. Let C^* be a plane containing C, parallel to $\mathcal{U}^*$ (see Figure 1). The random spheres, whose centers are located between $\mathcal{U}^*$ and $\mathcal{W}^*$ cast shadows on C. As stated in the Introduction, the objective is to develop formulae for the computation of the probabilities of simultaneous visibility of any n points on C $(n \geq 1)$. These probabilities are then applied to determine the moments of total visibility measures on C, and to approximate its distribution. In the present section we discuss the basic stochastic and geometric structure of the field.

Let u^*, w^* and r^* be the distances of $\mathcal{U}^*$, $\mathcal{W}^*$ and C^* from $\underset{\smile}{O}$, respectively. It is assumed that

$$0 < b \leq u^* < w^* < r^* - b, \tag{2.1}$$

where b is the maximal radius of a sphere. Let $\mathcal{M}$ be a plane passing through $\underset{\smile}{O}$ and C. Let $\mathcal{U}$ and $\mathcal{W}$ be straight lines at which $\mathcal{M}$ interesects $\mathcal{U}^*$ and $\mathcal{W}^*$. The three parallel

References

[1] Dwight, H.B. *Tables of Integrals and Other Mathematical Tables*, The McMillan Co. (4th Edition), 1964.

[2] Yadin, M. and Zacks, S. (1982) Random coverage of a circle with applications to a shadowing problem, *J. Appl. Prob.*, 19: 562-577.

[3] Yadin, M. and Zacks, S. (1984) The distribution of the random lighted portion of a curve in a plane shadowed by a Poisson random field of obstacles. *Statistical Signal Processing*, E.J. Wegman and J.B. Smith, Ed. Marcel Dekker, N.Y.

[4] Yadin, M. and Zacks, S. (1985) The visibility of stationary and moving targets in the plane subject to a Poisson field of shadowing elements, *J. App. Prob.*, 22: 776-786.

[5] Yadin, M. and Zacks, S. (1982) The Distributions of measures of visibility on line segments in three dimensional spaces under Poisson shadowing processes. Technical Report No. 4, Contract N00014-81-K-0407 (NR 042-276) with the Office of Naval Research.

[6] Yadin, M. and Zacks, S. (1986) Discretization of a Semi-Markov Shadowing Process, Technical Report No. 2, Contract DAAG29-84-K-0191 with the U.S. Army Research Office.